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LETTER TO THE EDITOR

Measurement of foam drainage using AC conductivity

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Abstract. AC conductivity measurements have been used to determine the local liquid fraction in a foam column under conditions of free and forced drainage. The data can be fitted very well with the theory of Verbist and Weaire.

Foam drainage plays an important part in the formation and evolution of liquid foams. Recent theoretical studies by Verbist and Weaire describe the main features of both free drainage [1, 2], where liquid drains out of a foam due to gravity, and forced drainage [3], where liquid is introduced to the top of a column of foam. In the latter case a solitary wave of constant velocity is generated when liquid is added at a constant rate [4]. In this paper we present preliminary results for both scenarios using a simple conductimetric method which provides a measure of the local liquid fraction Φ_l of the foam. These results show good agreement with the theoretical predictions.

When taken together with parallel work [1, 6] using other techniques, this agreement adds to our confidence that the proposed model for drainage has a wide regime of validity. This is not self-evident, since it is based on a number of idealizations and approximations. None of these is essentially new. Nevertheless it does not seem to have been realized that, when taken together without further elaboration, they present us with a very simple yet realistic theory [3], whose qualitative predictions are transparent and are confirmed by experiment. Forced foam drainage may well be the best prototype for certain general phenomena described by non-linear differential equations, particularly the type of solitary wave which is most familiar in tidal bores. In addition, we now have a firm base on which to develop more extensive theories which incorporate additional effects, as required by experiment whenever the model fails.

The model developed by Verbist and Weaire idealizes the network of Plateau borders, through which the majority of liquid is assumed to drain, as a set of N identical pipes of cross section A, which is a function of position and time. The following non-linear partial differential equation [3] is derived for A:

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x}\left(A^2 - \frac{\sqrt{A}}{2}\frac{\partial A}{\partial x}\right) = 0 \qquad (1)$$

where x and t are scaled position and time coordinates respectively [3].

In the case of forced drainage, appropriate boundary conditions result in a solitary-wave solution of the form

$$A(x, t) = \begin{cases} v \tanh^2(\sqrt{v}[x - vt]) & x \leqslant vt \\ 0 & x \geqslant vt \end{cases} \qquad (2)$$

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where v is the velocity of the wavefront.

For the case of free drainage, the initial liquid fraction may be taken to be constant throughout the foam and a condition of zero flow at the top of the foam is imposed, giving an implicit solution [2]:

$$A^{3/2} - \frac{x}{2(t+t_a)}A^{1/2} - \frac{1}{6(t+t_a)} = 0 \tag{3}$$

where t_a is an integration constant. The preliminary measurements presented here were intended to check the validity of both of these predictions.

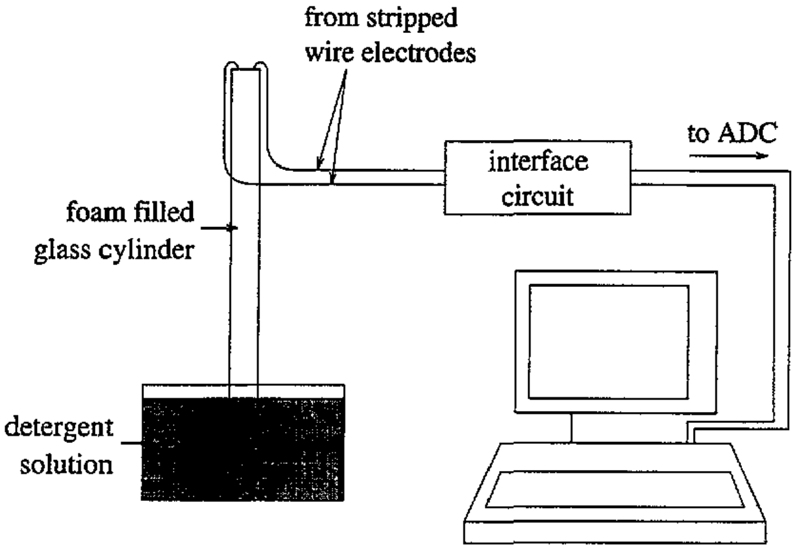

Figure 1. Schematic diagram of the experimental set-up.

Figure 1 shows an overview of the experimental set-up. A magnified view of the electrode arrangement is shown in figure 2. The electrode wires are stretched tightly and fixed to the outside of the tube. A foam is created in the tube using the method outlined in [4], that is, N_2 is blown through a fine glass nozzle into a bath of detergent solution. The measurements reported are for tube diameter 15 mm, bubble diameter 1.9 mm, for the free-drainage experiment and bubble diameter 1.7 mm for the forced-drainage experiment. In order to avoid electrode polarization in the foam, AC measurements are made using the analogue-to-digital converter of a microcomputer.

The electrical properties of foams were previously studied by Clark [5] who found that a linear relationship existed between the conductivity and liquid fraction of a foam. This was confirmed by our own measurements with liquid fraction estimated by comparing the height of submerged foam to the total column height. The apparatus was recalibrated for each individual foam.

To produce a solitary wave, the foam is allowed to drain until it is close to equilibrium. Solution is then added at the top of the cylinder at a constant rate using either a burette or a peristaltic pump. Figure 3 shows an example of a solitary wave measured in this way.

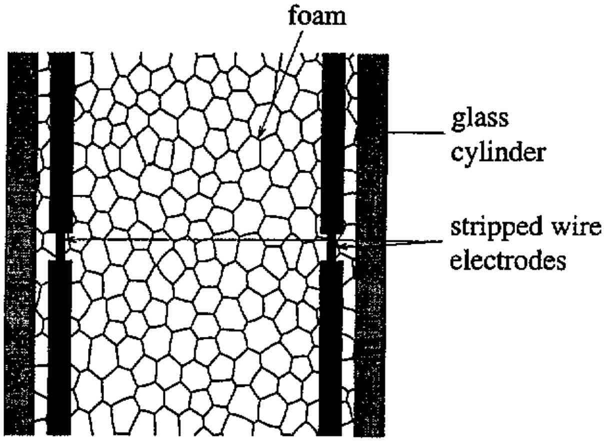

Figure 2. Magnified view of the stripped wire electrode arrangement.

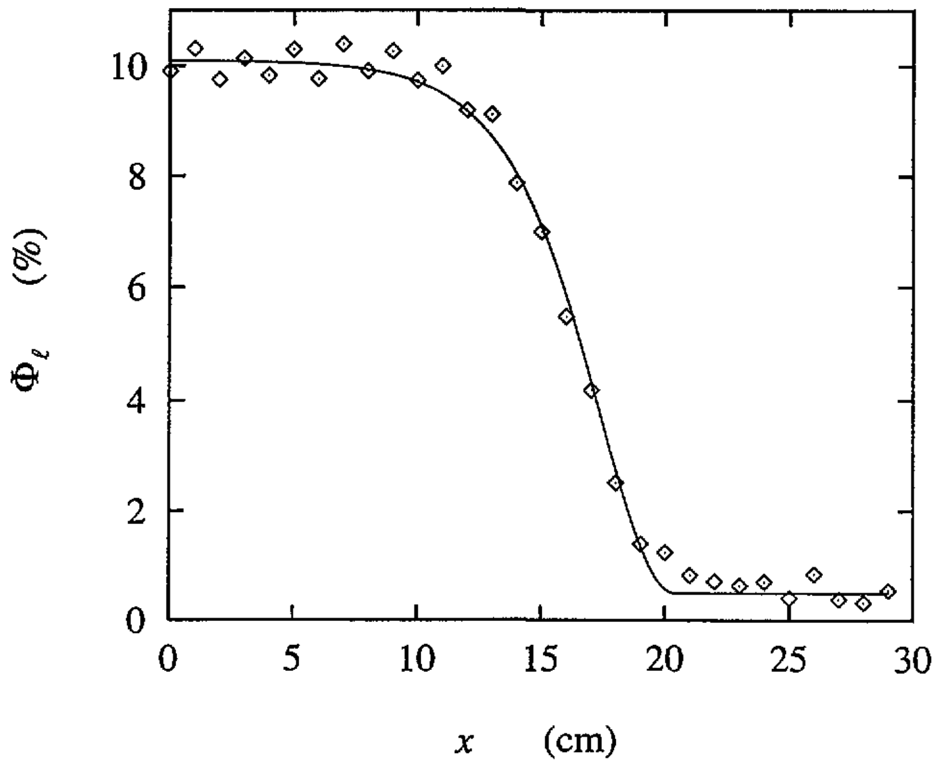

Figure 3. Foam profile for the forced-drainage experiment. The fitted line is of the form of equation (4).

In the free-drainage experiment solution is introduced until a steady state is reached over the entire column. The flow is then stopped. Figure 4 shows an example of a free-drainage curve. The initial time ($T = 0$) is that at which free drainage begins.

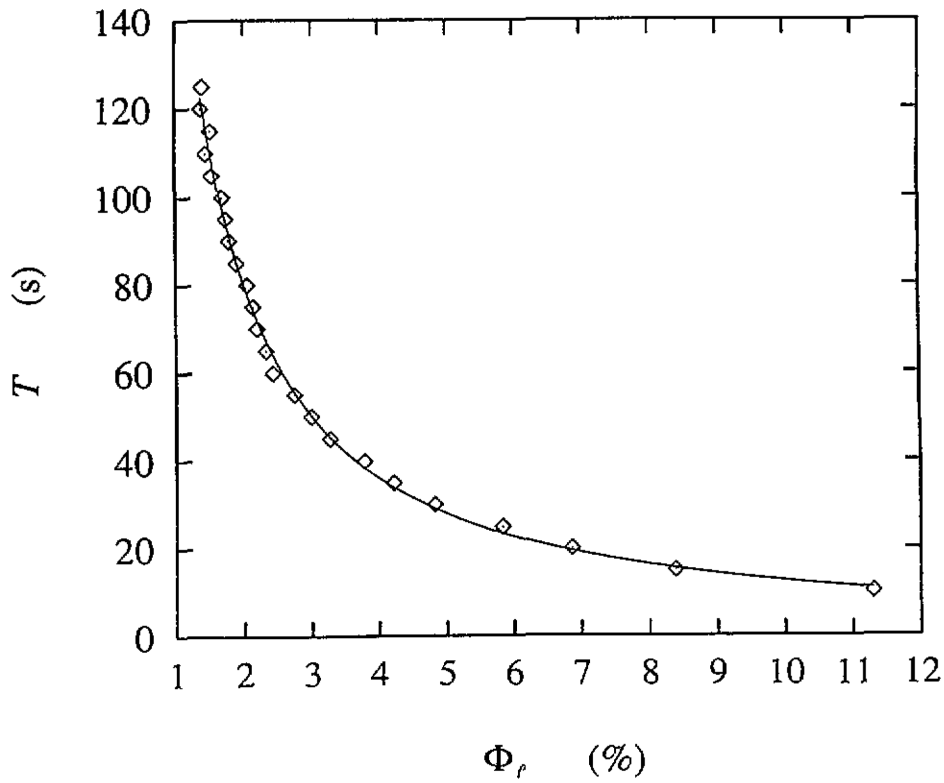

Figure 4. Decay of the liquid fraction with time for a typical free-drainage experiment. The fitted line is of the form of equation (7). Note that time is presented on the vertical axis due to the implicit nature of equation (7).

In the spirit of [1], [2] and [3] the area A defined above is assumed to be proportional to the liquid fraction Φ_l, which should be accurate for relatively low values of Φ_l. Accordingly we have used the following formula to fit the forced-drainage profile:

$$\Phi_l(x) = \begin{cases} a \tanh^2([x - x_0]/w) + b & x \leqslant Vt \\ 0 & x \geqslant Vt. \end{cases} \tag{4}$$

This corresponds to equation (2), with the addition of a constant b, which is necessary because the initial equilibrium state has a finite liquid fraction, whereas the solitary-wave profile corresponds to the idealized case of the wetting of a completely dry foam. Note that w represents the width of the wavefront, x_0 is the starting position of the wave and a is a scaling factor. Here the measured liquid fraction at a fixed point is converted into a spatial profile according to $x = Vt$ where V is the observed solitary-wave velocity. The line through the data in figure 3 was fitted by eye, using the above form, and adjusting w, x_0, a and b.

Using $A \propto \Phi_l$ for the free-drainage case, we find from equation (3)

$$T + T_a = \frac{\mu X}{2\lambda C^2}\Phi_l^{-1} + \frac{1}{6\lambda C^2}\Phi_l^{-3/2} \tag{5}$$

where $A = C^2\Phi_l$, T is measured time given by $t = \lambda T$ and X is measured distance given by $x = \mu X$. At $t = 0$ and $x = 0$ it can be shown that

$$T_a = \frac{1}{6\lambda C^2}\Phi_{l0}^{-3/2} \tag{6}$$

where Φ_{l0} is the liquid fraction at $t = 0$. Substituting into equation (5) gives

$$T = \frac{k}{\Phi_l} + T_a\left(\left(\frac{\Phi_{l0}}{\Phi_l}\right)^{3/2} - 1\right) \tag{7}$$

where $k = \mu X/2\lambda C^2$.

The line fitted to the data in figure 4 is of the form of equation (7). To analyse the free-drainage data further, we can define a function $F(\Phi_l)$ equal to the right-hand side of equation (7). Using fitted values of k and T_a and empirical values of Φ_{l0} (ranging from 6% to 18%) we have transformed drainage data for a family of drainage curves obtained from a single foam. Figure 5 shows $F(\Phi_l)$ plotted against T for these data. The data are closely grouped around the line of unit slope and zero intercept.

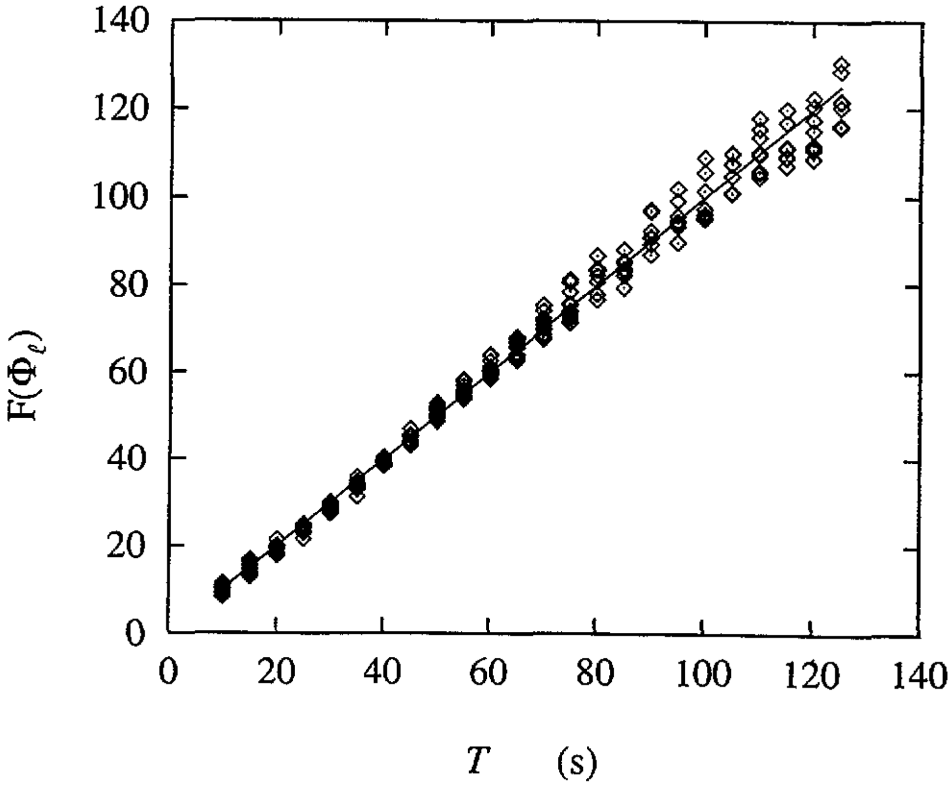

Figure 5. $F(\Phi_l)$ plotted against time for a set of eight free-drainage curves measured from a single foam. According to theory, the data should all fall on the line of unit slope and zero intercept which is shown.

In conclusion, we have succeeded in providing confirmation of the main points of the drainage theory of Verbist and Weaire although a more quantitative test is desirable. In particular there is a need to relate the fitting parameters for both forced- and free-drainage curves to physical quantities such as surface tension and foam viscosity which characterize the solution used. It would appear that conductivity provides a very convenient technique for such further tests.

Alternatively, a capacitance method for the measurement of liquid fraction of non-conducting foams has been developed by Hutzler *et al.* Results for solitary-wave phenomena described in [3] will be presented in a future paper [6].

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Note added in proof. It has come to our attention that the model equation (3) and its solitary wave solution were previously found by I I Gol'dfarb, K B Kann and I R Shreiber: see *Fluid Dynamics* **23** 244 (1988).

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